A Coq mechanised formal semantics for real life SQL queries

Formally reconciling SQL and bag relational algebra

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LRI - CNRS - Université Paris Sud

Coq Workshop@Floc18 Oxford 8th of July 2018

Motivations

Data are pervasive and valuable ...

... Little attention to guarantee systems are reliable and safe.

How to obtain strong guarantees?

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Strong guarantees : proof assistants, Coq or Isabelle

Datacert project 2012-???

Coq formalisation of data-centric languages and systems

Relational database systems

Most widespread systems

Underlying theory is well known [Codd70]

One standard SQL *the* relational database programming language

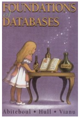
Mature implementations

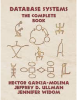
Oracle, DB_2 IBM, SQLServer, Postgresql, MySql, SQLite ...

Mid term goal: provide a Coq verified SQL's compiler

Sources (methodology) and goals

Foundations (studying) relational model and algebra





ANSI/ISO Standard (reading)

1500 pages natural language ... SQL's description

Mainstream systems, Postgresql and Oracle$^{\text{TM}}$ (testing)

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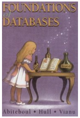

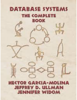

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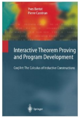
1500 pages natural language ... SQL's description

Mainstream systems, Postgresql and Oracle™ (testing)

Reconciling all of them

with strongest possible correctness guarantees (Coq)



Relational model and algebra

- Information modeling:

through relations and tuples

Structure: relation name and sort (finite set of attributes)

r(a, b) relation name r sort: {a,b}

- Information extraction:

through query languages (relational algebra)

Relational model and algebra

Two perspectives:

unnamed	vs	named
$r = \{(1, 2); (3, 2); (1, 1)\}$		$r = \{t1; t2; t3\}$

$$t1(a) = 1,\ t1(b) = 2$$
$$t2(a) = 3,\ t2(b) = 2$$
$$t3(a) = 1,\ t3(b) = 1$$

Relational model and algebra

Two perspectives:

unnamed	vs	named
$r = \{(1, 2); (3, 2); (1, 1)\}$		$r = \{t1; t2; t3\}$

$$\begin{aligned} t1.a &= 1, \; t1.b = 2 \\ t2.a &= 3, \; t2.b = 2 \\ t3.a &= 1, \; t3.b = 1 \end{aligned}$$

Relational model and algebra

unamed (SPC)

$$
\begin{array}{rcl}
q & := & r \mid \sigma_f(q) \mid \pi_W(q) \mid q \times q \\
  & \mid & q \cup q \mid q \cap q \mid q \setminus q
\end{array}
$$

named (SPJR)

$$
\begin{array}{rcl}
q & := & r \mid \sigma_f(q) \mid \pi_W(q) \mid \rho_g(q) \mid q \bowtie q \\
  & \mid & q \cup q \mid q \cap q \mid q \setminus q
\end{array}
$$

Relational model and algebra

- $\sigma_f(q) = \{t \in q \mid f(t)\}$
- $\pi_W(q) = \{t|_W \mid t \in q\}$
- $q_1 \bowtie q_2 = \{t \mid \exists t_1 \in q_1, \exists t_2 \in q_2, t|_{sort(q_1)} = t_1 \wedge t|_{sort(q_2)} = t_2\}$

- $\rho_g(q) = \{t' \mid \exists t \in q, \forall a \in sort(q), t'.g(a) = t.a\}$

Relational model and algebra

- $\sigma_f(q) = \{t \in q \mid f(t)\}$
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Simple algebraic queries

Assuming `tbl1(a,b,c)` and `tbl2(d,e)`

$\pi_{\{\mathtt{a},\mathtt{c}\}}(\sigma_{\mathtt{b}>3}(\mathtt{tbl1}))$

$\rho_{\{\mathtt{a}\to\mathtt{a1};\mathtt{c}\to\mathtt{c1}\}}(\pi_{\{\mathtt{a},\mathtt{c}\}}(\sigma_{\mathtt{b}>3}(\mathtt{tbl1})))$

$\sigma_{\mathtt{b}=\mathtt{d}\wedge\mathtt{c}=\mathtt{e}}(\mathtt{tbl1} \bowtie \mathtt{tbl2})$

SQL: a simple declarative language

SQL "inter-galactic" dialect for manipulating (relational) data

Declarative DSL describe what opposed as how

```
select expression
from query
where condition
group by expression
having condition
```

With attribute's names as first-class citizens

$\Rightarrow$ name-based perspective

SQL's compilation

Syntactic analysis $SQL \rightarrow AST$

Semantic analysis $AST \rightarrow AST_{sem}$

Textbooks
: leaves = relations
: nodes = relational algebra operators

Real life depends on DB vendors

Optimisation / Query planning $AST_{sem} \rightarrow AST_{phys}$

Logical rewritings / algebraic equivalences

Physical
- auxiliary data structures (B trees, Hash tables etc)
- physical algebra – different implementations of operators
- data dependent statistics

SQL's compilation

Syntactic analysis $\text{SQL} \rightarrow \text{AST}$

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Real life: depends on DB vendors

Optimisation / Query planning $\text{AST}_{sem} \rightarrow \text{AST}_{phys}$

See Chantal Keller's talk at ITP !

Logical: rewritings / algebraic equivalences

Physical:
- auxiliary data structures (B trees, Hash tables etc)
- physical algebra – different implementations of operators
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SQL's compilation

This talk

Semantic analysis $\text{AST} \rightarrow \text{AST}_{sem}$

Textbooks	
	leaves = relations
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Providing a formal semantics to SQL

Formally relating SQL's semantics with a relevant algebra

SQL's compilation

This talk

Semantic analysis $AST \rightarrow AST_{sem}$

Textbooks
leaves = relations
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Real life depends on DB vendors

Providing a formal semantics to SQL

Formally relating SQL's semantics with a relevant algebra

30 years research efforts

[Ceri&al85, Negri&al91, Guagliardo&al17]

[Malecha&al10, Auerbach&al17, Chu&al17]

SQL: a Simple Declarative Language

Assuming `tbl1(a,b,c)` and `tbl2(d,e)`

```
select a, c from tbl1 where b>3;
```

$$\pi_{\{a,c\}}(\sigma_{b>3}(\texttt{tbl1}))$$

```
select a as a1, c as c1 from tbl1 where b>3;
```

$$\rho_{\{a\to a1; c\to c1\}}(\pi_{\{a,c\}}(\sigma_{b>3}(\texttt{tbl1})))$$

```
select * from tbl1, tbl2 where b=d and c=e;
```

$$\sigma_{b=d \wedge c=e}(\texttt{tbl1} \bowtie \texttt{tbl2})$$

```
select b, sum(a) from tbl1 where a= 7 group by b;
```

$$\gamma_{b, sum(a)}(\sigma_{a=7}(\texttt{tbl1}))$$

```
select b, 2*(a+c), sum(a) from tbl1 where a+b = 7 group by
 b, a+c having avg(b+c) > 6;
```

No corresponding expression in textbooks' algebras

SQL: a Simple Declarative Language

Declarative DSL = ... simple

intented not to be Turing complete

But

Not so simple ...

SQL: not so simple

Based on relational algebra for the `select-from-where` part

Mixes two algebras: the name based SPJR and the unnamed SPC

SQL: not so simple

Based on relational algebra for the `select-from-where` part

Mixes two algebras: the name based SPJR and the unnamed SPC

Quoting page 51 of the ISO document attributes are specified by:

"*The terms column, field, and attribute refer to structural components of tables, row types, and structured types, [...] in analogous fashion. As the structure of a table consists of one or more columns, so does the structure of a row type consist of one or more fields [...] Every structural element, whether a column, a field, or an attribute, is primarily a* **name paired** *with a declared* **type**. *The elements of a structure are* **ordered**. *Elements in different* **positions** *in the same structure can have the same declared type but* **not the same name**. *[...] in* **some circumstances** *[...] the compatibility [...] is determined solely by considering the declared types of each pair of elements at the same* **ordinal position**."

SQL: not so simple

Based on relational algebra for the `select-from-where` part

Mixes two algebras: the name based SPJR and the unnamed SPC

Enjoys a bag semantics

Manages complex expressions and aggregates

with `NULL` values
which represent incomplete information
handled by 3-valued logic with `unknown`

and nested, correlated queries

SQL: not so simple

Based on relational algebra for the `select-from-where` part

Mixes two algebras: the name based SPJR and the unnamed SPC

Enjoys a bag semantics

Manages complex expressions and aggregates

with `NULL` values
which represent incomplete information
handled by 3-valued logic with `unknown`

and nested, correlated queries
$\Longrightarrow$ strange behaviours

SQL: NULL's (I)

$$r = \{\!|(\mathtt{a}{=}1), (\mathtt{a}{=}\mathtt{NULL})|\!\}$$
$$s = \{\!|(\mathtt{a}{=}\mathtt{NULL})|\!\}$$
$$t = \{\!|(\mathtt{a}{=}1), (\mathtt{a}{=}\mathtt{NULL}), (\mathtt{a}{=}\mathtt{NULL})|\!\}$$

Q_1: `select r.a+2 as b from r;`

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Q_1:
```
select r.a+2 as b from r;
```

$$\{\!|\ (\mathtt{b}\ =\ 1{+}2); (\mathtt{b}\ =\ \mathtt{NULL}{+}2)\ |\!\}$$

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Q_1: `select r.a+2 as b from r;`

$$\{\!|\ (\mathtt{b}\ =\ 1{+}2); (\mathtt{b}\ =\ \mathtt{NULL}{+}2)\ |\!\}$$
$$\{\!|\ (\mathtt{b}\ =\ 3); (\mathtt{b}\ =\ \mathtt{NULL})\ |\!\}$$

NULL is an absorbing element

SQL: NULL's (II)

$$r = \{\!|(\mathrm{a}{=}1), (\mathrm{a}{=}\mathtt{NULL})|\!\}$$
$$s = \{\!|(\mathrm{a}{=}\mathtt{NULL})|\!\}$$
$$t = \{\!|(\mathrm{a}{=}1), (\mathrm{a}{=}\mathtt{NULL}), (\mathrm{a}{=}\mathtt{NULL})|\!\}$$

Q_2:
```
select r.a from r where r.a not in (select s.a from s);
```

Q_3:
```
select r.a from r where
     not exists (select * from s where s.a = r.a);
```

Q_4:
```
select r.a from r except select s.a from s;
```

SQL: NULL's (II)

$$r = \{\!|(a{=}1), (a{=}\mathtt{NULL})|\!\}$$
$$s = \{\!|(a{=}\mathtt{NULL})|\!\}$$
$$t = \{\!|(a{=}1), (a{=}\mathtt{NULL}), (a{=}\mathtt{NULL})|\!\}$$

Q_2: `select r.a from r where r.a not in (select s.a from s);`

$$\{t.a \mid t \in r \wedge \neg(t.a \in \{\ (a{=}\mathtt{NULL})\ \})\}$$
$$\{t.a \mid t \in r \wedge (t.a \neq \mathtt{NULL})\}$$

Q_3:
```
select r.a from r where
       not exists (select * from s where s.a = r.a);
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Q_4: `select r.a from r except select s.a from s;`

SQL: NULL's (II)

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$\{\!|\quad|\!\}$ 1 and NULL are not different from NULL

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`not exists (select * from s where s.a = r.a);`

Q_4: `select r.a from r except select s.a from s;`

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$$\{t.a \mid t \in r \wedge \{u \mid u \in s \wedge u.a = t.a\} = \emptyset\}$$

Q_4: `select r.a from r except select s.a from s;`

SQL: NULL's (II)

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$\{\!|\ (\mathrm{a}\ =\ 1); (\mathrm{a}\ =\ \mathtt{NULL})\ |\!\}$ NULL neither equals to anything else

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$\{\!|\ (\text{a} = 1); (\text{a} = \texttt{NULL})\ |\!\}$ NULL neither equals to anything else

Q_4: `select r.a from r except select s.a from s;`

$\{\ (\text{a} = 1)\ \}$ syntactic equality is used

SQL: NULL's (III)

$$r = \{\!|(\mathrm{a}{=}1), (\mathrm{a}{=}\texttt{NULL})|\!\}$$
$$s = \{\!|(\mathrm{a}{=}\texttt{NULL})|\!\}$$
$$t = \{\!|(\mathrm{a}{=}1), (\mathrm{a}{=}\texttt{NULL}), (\mathrm{a}{=}\texttt{NULL})|\!\}$$

Q_5: `select t.a, count( * ) as c from t group by t.a;`

SQL: NULL's (III)

$$r = \{\!|(\mathrm{a}{=}1), (\mathrm{a}{=}\texttt{NULL})|\!\}$$
$$s = \{\!|(\mathrm{a}{=}\texttt{NULL})|\!\}$$
$$t = \{\!|(\mathrm{a}{=}1), (\mathrm{a}{=}\texttt{NULL}), (\mathrm{a}{=}\texttt{NULL})|\!\}$$

Q_5: `select t.a, count( * ) as c from t group by t.a;`

$\{\!|$ (a = NULL, c = 2); (a = 1, c = 1) $|\!\}$

NULL equals NULL for grouping

SQL: aggregates, nesting, correlated queries

Designing instances and queries for testing against Postgresql and Oracle™

t_1

a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1
1	1	2	1	3	1		
1	2	2	2	3	2		
1	3	2	3	3	3		
1	4	2	4	3	4		
1	5	2	5	3	5		
1	6	2	6			4	6
1	7	2	7			4	7
1	8	2	8			4	8
1	9	2	9			4	9
1	10	2	10			4	10

t_2

a_2	b_2
7	7
7	7

SQL: aggregates, nesting, correlated queries

$$
\begin{array}{rcl}
t_1 & = & \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
 & & \{\!|(a1 = 2, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
 & & \{\!|(a1 = 3, b1 = i) \mid 1 \leq i \leq 5|\!\} \cup \\
 & & \{\!|(a1 = 4, b1 = i) \mid 6 \leq i \leq 10|\!\} \cup \\
 & & \\
t_2 & = & \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
\end{array}
$$

Q6: `select a1, max(b1) from t1 group by a1;`

SQL: aggregates, nesting, correlated queries

$$
\begin{array}{rcl}
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 & & \{\!|(a1 = 3, b1 = i) \mid 1 \leq i \leq 5|\!\} \cup \\
 & & \{\!|(a1 = 4, b1 = i) \mid 6 \leq i \leq 10|\!\} \cup \\
 & & \\
t_2 & = & \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
\end{array}
$$

Q6: `select a1, max(b1) from t1 group by a1;`

$$
\left\{\!\!\left| \begin{array}{l} (a1 = 1, max = 10); (a1 = 2, max = 10); \\ (a1 = 3, max = 5); (a1 = 4, max = 10) \end{array} \right|\!\!\right\}
$$

SQL: aggregates, nesting, correlated queries

$$
\begin{aligned}
t_1 &= \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
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&\quad \{\!|(a1 = 4, b1 = i) \mid 6 \leq i \leq 10|\!\} \cup
\end{aligned}
$$

$$
t_2 = \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
$$

Q7:
```
select a1 from t1 group by a1 having
         exists (select a2 from t2 group by a2
                   having sum(1+0*a2) = 2);
```

`sum(1+0*a2)`: computes the number of tuples in a group

SQL: aggregates, nesting, correlated queries

$$
\begin{array}{rcl}
t_1 & = & \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
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\end{array}
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$$
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which group?

SQL: aggregates, nesting, correlated queries

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$$
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`sum(1+0*a2)`: computes the number of tuples in a group

which group?

$$
\{\!| \ (a1 = 1); (a1 = 2); (a1 = 3); (a1 = 4) \ |\!\}
$$

SQL: aggregates, nesting, correlated queries

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\begin{aligned}
t_1 &= \{\!|(a1 = 1, b1 = i) \mid 1 \le i \le 10|\!\} \cup \\
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\end{aligned}
$$

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Q7:
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select a1 from t1 group by a1 having
        exists (select a2 from t2 group by a2
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```

`sum(1+0*a2)`: computes the number of tuples in a group

which group?

$$
\{\!| \ (a1 = 1); (a1 = 2); (a1 = 3); (a1 = 4) \ |\!\}
$$

`sum(1+0*a2)` is evaluated to 2 for each a2-group

SQL: aggregates, nesting, correlated queries

$$
\begin{aligned}
t_1 \quad = \quad & \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
& \{\!|(a1 = 2, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
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\end{aligned}
$$

$$
t_2 \quad = \quad \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
$$

Q8:
```
select a1 from t1 group by a1 having
          exists (select a2 from t2 group by a2
                         having sum(1+0*a2) = 10);
```

SQL: aggregates, nesting, correlated queries

$$
\begin{array}{rcl}
t_1 & = & \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
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t_2 = \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
$$

Q8:
```
select a1 from t1 group by a1 having
         exists (select a2 from t2 group by a2
                 having sum(1+0*a2) = 10);
```

$$
\{\!| \quad |\!\}
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SQL: aggregates, nesting, correlated queries

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```

$$\{\!|\quad|\!\}$$

`sum(1+0*a2)` is evaluated to $2 \neq 10$ for each `a2`-group

SQL: aggregates, nesting, correlated queries

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\begin{aligned}
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Q9(k):

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select a1 from t1 group by a1 having
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$$

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SQL: aggregates, nesting, correlated queries

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`sum(1)` is evaluated to 2 for each a2-group

SQL: aggregates, nesting, correlated queries

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`sum(1)` is evaluated to 2 for each `a2`-group

Tentative conclusion: `1+0*a2 = 1`

SQL: aggregates, nesting, correlated queries

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\begin{array}{rcl}
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t_2 = \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
$$

Q10:
```
select a1 from t1 group by a1 having
          exists (select a2 from t2 group by a2
                       having sum(1+0*a1) = 10);
```

SQL: aggregates, nesting, correlated queries

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\begin{array}{rcl}
t_1 & = & \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\}\cup \\
 & & \{\!|(a1 = 2, b1 = i) \mid 1 \leq i \leq 10|\!\}\cup \\
 & & \{\!|(a1 = 3, b1 = i) \mid 1 \leq i \leq 5|\!\}\cup \\
 & & \{\!|(a1 = 4, b1 = i) \mid 6 \leq i \leq 10|\!\}\cup
\end{array}
$$

$$
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\{\!|\ (a1 = 1); (a1 = 2)\ |\!\}
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SQL: aggregates, nesting, correlated queries

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```

$$
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$$

`sum(1+0*a1)` is evaluated for each `a1`-group

Conclusion: `1 <> 1+0*a1` in some contexts (since Q9(10) $\neq$ Q10)

SQL: aggregates, nesting, correlated queries

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\begin{aligned}
t_1 \quad = \quad & \{\!|(a1 = 1, b1 = i) \mid 1 \leq i \leq 10|\!\} \cup \\
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$$

Q11(k):

```
select a1 from t1 group by a1 having
    exists (select a2 from t2 group by a2
                having sum(1+0*a1)+sum(1+0*a2) = k);
```

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```

$$
k = 7 \rightsquigarrow \{\!|\ (a1 = 3); (a1 = 4)\ |\!\} \quad k = 12 \rightsquigarrow \{\!|\ (a1 = 1); (a1 = 2)\ |\!\}
$$

$$
k \neq 7, k \neq 12 \rightsquigarrow \{\!|\quad|\!\}
$$

SQL: aggregates, nesting, correlated queries

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$k = 7 \leadsto \{\!|\ (a1 = 3); (a1 = 4)\ |\!\}$ $\quad k = 12 \leadsto \{\!|\ (a1 = 1); (a1 = 2)\ |\!\}$

$$k \neq 7, k \neq 12 \leadsto \{\!|\quad|\!\}$$

Different sub-expressions of same expression are evaluated
in different environments

SQL: aggregates, nesting, correlated queries

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Q12(k):

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select a1 from t1 group by a1 having
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\end{array}
$$

$$
t_2 = \{\!|(a2 = 7, b2 = 7), (a2 = 7, b2 = 7)|\!\}
$$

Q14:
```
select a1 from t1 group by a1 having
     exists (select a2 from t2 group by a2
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SQL: aggregates, nesting, correlated queries

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\begin{aligned}
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```
select a1 from t1 group by a1 having
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```

```
ERROR: subquery uses ungrouped column "t1.b1" from outer query
LINE 1: ...sts (select a2 from t2 group by a2 having sum(1+0*b1+0*b2) =
```

SQL: aggregates, nesting, correlated queries

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$$

Q15:
```
select a1 from t1 group by a1 having
      exists (select a2 from t2 group by a2
                    having sum(1+0*b1+0*a2) = 12);
```

SQL: aggregates, nesting, correlated queries

$$
\begin{aligned}
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ERROR: subquery uses ungrouped column "t1.b1" from outer query
LINE 1: ...sts (select a2 from t2 group by a2 having sum(1+0*b1+0*a2) =
```

Environments

A stack of slices, nesting levels, innermost on top

(attributes, grouping expressions, group of tuples)

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Environments

A stack of slices, nesting levels, innermost on top

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- simple expression $\rightsquigarrow$ get the (unique) binding of each attribute
- complex expression `function`$(\overline{e})$
  $\rightsquigarrow$ evaluate independently each e_i of $(\overline{e})$
- complex expression `aggregate(cst)`
  $\rightsquigarrow$ use innermost slice (cardinality)
- complex expression `aggregate`(e) in $[S_n; \ldots; S_1]$
  $\rightsquigarrow$ find the smallest "suitable" suffix $[S_{i+1}; S_i, ; \ldots; S_1]$
  s.t. e is built upon $A(S_{i+1}) \cup G(S_i) \cup \ldots \cup G(S_1)$
  $\rightsquigarrow$ split tuples of $(i+1)$th slice
  $\rightsquigarrow [(A(S_{i+1}), G(S_{i+1}), [t_{i+1}]); S_i; \ldots; S_1]\ t_{i+1} \in T(S_{i+1})$

Environments

A stack of slices, nesting levels, innermost on top

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Evaluation

- simple expression ⇝ get the (unique) binding of each attribute
- complex expression function($\bar{e}$)
  ⇝ evaluate independently each e of ($\bar{e}$)
- complex expression aggregate(cst)
  ⇝ use innermost slice (cardinality)
- complex expression aggregate(e) in $[S_n; \ldots; S_1]$
  ⇝ find the smallest "suitable" suffix $[S_{i+1}; S_i, ; \ldots; S_1]$
  s.t. e is built upon $A(S_{i+1}) \cup G(S_i) \cup \ldots \cup G(S_1)$
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SQHeLL

SQL$_{Coq}$ Queries

```
Inductive set_op := Union | Intersect | Except.
Inductive select := Select_As : aggterm → attribute → select.
Inductive select_item := Select_Star | Select_List : list select → select_item.
Inductive group_by := Finest_P | Group_By : list funterm → group_by.

Inductive att_renaming :=
  Att_As : attribute → attribute → att_renaming.
Inductive att_renaming_item :=
   Att_Ren_Star | Att_Ren_List : list att_renaming → att_renaming_item.

Inductive sql_query :=
  | Table : relname → sql_query
  | Set : set_op → sql_query → sql_query → sql_query
  | Select :
      (** select *) select_item  →
      (** from *) list from_item →
      (** where *) formula sql_query →
      (** group by *) group_by →
      (** having *) formula sql_query → sql_query

with from_item := From_Item : sql_query → att_renaming_item → sql_from_item.
```

no optional `where`, `group by`, nor `having`

no `where` ⇝ `TTrue`

no `group by` but `having` ⇝ `Group_By nil`

no `group by` nor `having` ⇝ `Finest_P`+ `TTrue`

SQL_{Coq} Formulas

```
Inductive conjunct := And | Or.
Inductive quantifier := All | Any.

Inductive formula (dom : Type) :=
  | Conj : conjunct → formula dom → formula dom → formula dom
  | Not : formula dom → formula dom
  | TTrue : formula dom
  | Pred : predicate → list aggterm → formula dom
  | Quant : list aggterm → predicate → quantifier → dom → formula dom
  | In : list select → dom → formula dom
  | Exists : dom → formula dom.
```

almost FO + `in` + `exists`

$\forall \rightsquigarrow$ `all`

$\exists \rightsquigarrow$ `any`

`in` (membership) $\rightsquigarrow$ $\_ \in \_$ (not a usual predicate over values)

`exists` $\rightsquigarrow$ non-emptiness test

parameterised by `dom`

intended to be a finite domain of interpretation

Coq mechanised semantics

Simple expressions

```
(* The type of evaluation environments *)
Definition env_type := list (list attribute * group_by * list tuple).

(* get the (unique) binding of each attribute *)
Fixpoint interp_dot env (a : attribute) :=
  match env with
    | nil ⇒ default_value a
    | (sa, gb, nil) :: env' ⇒ interp_dot env' a
    | (sa, gb, t :: l) :: env' ⇒ if a inS? labels t then (dot t a) else interp_dot env' a
   end.

Fixpoint interp_funterm  env t :=
  match t with
    | F_Constant c ⇒ c
    | F_Dot a ⇒ interp_dot env a
    | F_Expr f l ⇒ interp_symb f (List.map (fun x ⇒ interp_funterm env x) l)
  end.
```

Coq mechanised semantics

Complex expressions, environments

```
Fixpoint is_built_upon G f :=
  match f with
    | F_Constant _ ⇒ true
    | F_Dot _ ⇒ f inS? g
    | F_Expr s l ⇒ (f ins? G) || forallb (is_built_upon G) l
  end.

Definition is_a_suitable_env la env f :=
  is_built_upon
     (map (fun a ⇒ F_Dot a) la ++
        flat_map (fun slc ⇒ match slc with (_, G, _) ⇒ G end) env) f.

Fixpoint find_eval_env env f :=
  match env with
    | nil ⇒ if is_built_upon nil f then Some nil else None
    | (la1, g1, l1) :: env' ⇒
      match find_eval_env env' f with
        | Some _ as e ⇒ e
        | None ⇒ if is_a_suitable_env la1 env' f then Some env else None
      end
  end.
```

simply models SqHeLL, beginning

Coq mechanised semantics

Complex expressions, environments

```
Fixpoint interp_aggterm env (ag : aggterm) := match ag with
  | A_Expr ft ⇒ (* simple expression without aggregate *)
                interp_funterm env ft
  | A_fun f lag ⇒
    (** simple recursive call in order to evaluate independently the sub-expressions
       when the top symbol is a function *)
     interp_symb f (List.map (fun x ⇒ interp_aggterm env x) lag)
  | A_agg ag ft ⇒
    let env' := if is_empty (att_of_funterm ft)
                then (** the expression under ag is a constant *)
                     Some env
                else (** find the outermost suitable level *)
                     find_eval_env env ft in
    let lenv :=
        match env' with
          | None | Some nil ⇒
              (** this case should not happen for well-formed queries *) nil
          | Some ((la1, g1, l1) :: env'') ⇒
              (** the outermost group is split into *)
               map (fun t1 ⇒ (la1, g1, t1 :: nil) :: env'') l1
        end in
        interp_aggregate ag (List.map (fun e ⇒ interp_funterm e ft) lenv)
  end.
```

simply models SqHeLL, end
irrelevant cases (ill-formed queries) due to totality

Coq mechanised semantics

parametric Booleans and 3-valued logic

```
Module Bool. (* parametric Booleans *)
Record Rcd : Type := mk_R {
      b : Type;
      true : b;
      false : b;
      andb : b → b → b;
      orb : b → b → b;
      negb : b → b;
      [...]
      true_is_true : ∀b, is_true b = Datatypes.true ↔ b = true }.
End Bool.

Definition Bool2 : Bool.Rcd.
split with bool true false andb orb negb [...]

Inductive bool3 : Type := true3 | false3 | unknown3.
Definition andb3 b1 b2 := [...]
Definition orb3 b1 b2 := [...]
Definition negb3 b := [...]

Definition Bool3 : Bool.Rcd.
 split with bool3 true3 false3 andb3 orb3 negb3 [...]
```

interpretation of formulas parameterised by a Booleans,
⇝ 2-valued logic or 3-valued logic

NULLs ⇝ 3-valued logic

Coq mechanised semantics

Formulas

```
Hypothesis B : Bool.Rcd. (* parametric Booleans *)
Hypothesis I : env_type → dom → bagT (* bags of tuples *).
Fixpoint eval_formula env f : Bool.b B := match f with
  | Conj a f1 f2 ⇒ (interp_conj B a) (eval_formula env f1) (eval_formula env f2)
  | Not f ⇒ Bool.negb B (eval_formula env f)
  | TTrue ⇒ Bool.true B
  | Pred p l ⇒ interp_predicate p (map (interp_aggterm env) l)

  | Quant qtf p l sq ⇒ let lt := map (interp_aggterm env) l in
     interp_quant B qtf (fun x ⇒ let la := Fset.elements _ (labels T x) in
                                  interp_predicate p (lt ++ map (dot T x) la))
                 (Febag.elements _ (I env sq))

  | In s sq ⇒ let p := (projection env (Select_List s)) in
     interp_quant B Any
       (fun x ⇒ match Oeset.compare (OTuple T) p x with
         | Eq ⇒ if contains_null p then unknown else Bool.true B
         | _ ⇒ if (contains_null p || contains_null x) then unknown else Bool.false B end)
       (Febag.elements _ (I env sq))

  | Exists sq ⇒ if Febag.is_empty _ (I env sq) then Bool.false B else Bool.true B
 end.
```

evaluation parameterised by Booleans

subtleties in `In` for handling equality for `NULL`s:

`unknown` may be `unknown3` or `false`

Coq mechanised semantics

Queries

```
Fixpoint eval_sql_query env (sq : sql_query) {struct sq} :=
match sq with
 | Sql_Table tbl ⇒ instance tbl
 | Sql_Set o sq1 sq2 ⇒ [...]
 | Sql_Select s lsq f1 gby f2  ⇒
  let elsq := (** evaluation of the from part *)
        List.map (eval_sql_from_item env) lsq in
  let cc := (** selection of the from part by the formula f1, with old names *)
     Febag.filter _
         (fun t ⇒ Bool.is_true B (* casting parametric Booleans to Bool2 *)
                     (eval_sql_formula eval_sql_query (env_t env t) f1))
            (N_product_bag elsq) in
  (** computation of the groups grouped according to gby *)
  let lg1 := make_groups env cc gby in
  (** discarding groups according the having clause f2 *)
  let lg2 :=
    List.filter
      (fun g  ⇒ Bool.is_true B (* casting parametric Booleans to Bool2 *)
                    (eval_sql_formula eval_sql_query (env_g env gby g) f2))
      lg1 in
  (** applying the outermost projection and renaming, the select part s *)
   Febag.mk_bag BTupleT
     (List.map (fun g ⇒ projection (env_g env gby g) s) lg2)
  end
```

Empirical assessment

Executable semantics ⇝ checked against Postgresql and Oracle™

Previous queries and similar ones (up to 4 levels of nesting)

Random instance generator, 5 parameters: number of tables, number of attributes for each table, max size of a relation's instance, max integer value in relations' instances, proportion of `NULL`'s in instances,

Random query generator, 5 parameters: proportion of constants among expressions, max number of expressions in `select`, max number of queries in `from`, max number of grouping expressions in `group by`, max level of nesting

Relating SQL_{Coq} with an algebra

Define SQL_{Alg}

Extended relational algebra

Enjoying a bag semantics and

Natively accounting for `group by having`

Hence recovering well known algebraic equivalences

SQL_{Alg}, a Coq mechanised algebra

```
Inductive alg_query : Type :=
  | Q_Empty_Tuple : alg_query
  | Q_Table : relname → alg_query
  | Q_Set : set_op → alg_query → alg_query → alg_query
  | Q_Join : alg_query → alg_query → alg_query
  | Q_Pi : list select → alg_query → alg_query
  | Q_Sigma : (formula alg_query) → alg_query → alg_query
  (* extending the usual γ textbook operator *)
  | Q_Gamma :
      (* aggregated (output) expressions *) list select →
      (* grouping expressions *) list funterm →
      (* handling having condition *) (formula alg_query) →
      (* query *) alg_query → alg_query.
```

usual relational algebra + a generalized γ operator: `Q_Gamma`

`formula`s are shared with SQL_{Coq}

SQL$_{Alg}$'s mechanised semantics

```
Fixpoint eval_alg_query env q {struct q} : bagT :=
  match q with
  | Q_Empty_Tuple ⇒ Febag.singleton _ (empty_tuple T)
  | Q_Table r ⇒ instance r
  | Q_Set o q1 q2 ⇒ [...]
  | Q_Join q1 q2 ⇒ natural_join (eval_alg_query env q1) (eval_alg_query env q2)
  | Q_Pi s q ⇒
    Febag.map _ _
      (fun t ⇒ projection (env_t env t) (Select_List s)) (eval_alg_query env q)
  | Q_Sigma f q ⇒
    Febag.filter _
      (fun t ⇒ Bool.is_true B (eval_formula _ eval_alg_query (env_t env t) f))
      (eval_alg_query env q)
  | Q_Gamma s lf f q ⇒
    Febag.mk_bag _
      (map (fun l ⇒ projection (env_g env (Group_By lf) l) (Select_List s))
      (filter (fun l ⇒ Bool.is_true B
                 (eval_formula _ eval_alg_query (env_g env (Group_By lf) l) f))
              (make_groups env (eval_alg_query env q) (Group_By lf))))
  end.
```

environments and `formula`'s evaluation are shared with SQL$_{Coq}$

Equivalence

$$SQL_{Coq} \equiv SQL_{Alg}$$

From SQL_{Coq} to SQL_{Alg}

```
Fixpoint sql_query_to_alg basesort (sq : sql_query) :=
  match sq with
    | Sql_Table r ⇒ Q_Table r
    | Sql_Set o sq1 sq2 ⇒ [...]
    | Sql_Select s lsq f1 g f2 ⇒
        match s with
        | Select_Star ⇒ [...]
        | Select_List s ⇒
           let q1 := (** from clause is translated thanks to n-ary natural join *)
                        N_Q_Join (map sql_item_to_alg lsq) in
           let q2 := (** filtering against where condition *)
                        Q_Sigma (formula_to_alg f1) q1 in
        match g with
         | Finest_P ⇒
           (** no grouping, filtering against having condition, and then evaluation of select *)
               Q_Pi s (Q_Sigma (formula_to_alg f2) q2)
         | Group_By g ⇒
           (** grouping, using extended γ *)
               Q_Gamma s g (formula_to_alg f2) q2
        end
      end
  end with [...]
```

from $\rightsquigarrow$ $\bowtie$

Back translation from SQL_{Alg} to SQL_{Coq}

```
Hypothesis fresh (la : list attribute) : attribute.
Hypothesis fresh_is_fresh : ∀s, Oset.mem_bool (OAtt T) (fresh s) s = false.

Fixpoint alg_query_to_sql (q : alg_query) : sql_query :=
  match q with [...]
  | Q_Join q1 q2 ⇒
    let rho1 := (** fresh names for attributes of q1 *)  [...] in
    let rho2 := (** fresh names for attributes of q2 *) [...] in
    let rho1' := (** inverse of rho1 *)  [...] in
    let rho2' := (** inverse of rho2, over for attributes which do not belong to q1 *)  [...] in
    let f_join :=
      (* formula stating that new names for the same old shared attributes
         correspond to the same value : rho1(q1.a) = rho2(q2.a) *) [...] in
    Sql_Select (Select_List (rho1' ++ rho2'))
               (From_Item (alg_sql_query_to_sql q1) (Att_Ren_List rho1) ::
                From_Item (alg_sql_query_to_sql q2) (Att_Ren_List rho2) :: nil)
                      f_join Finest_P (Sql_True _)

  | Q_Pi s q ⇒ [...]
  | Q_Sigma f q ⇒ [...]

  | Q_Gamma s g h q ⇒
    Sql_Select (Select_List s) (From_Item (alg_query_to_sql q) Att_Ren_Star :: nil)
                (Sql_True _) (Group_By g) (alg_formula_to_sql h)
  end.
```

$\bowtie \rightsquigarrow$ from $\rightsquigarrow$ fresh names needed

Equivalence's theorems

```
Definition well_sorted_sql_table :=
  ∀tbl t, t inBE (instance tbl) → labels t =S= basesort tbl.

Fixpoint all_distinct lsa :=
  match lsa with
    | nil ⇒ true
    | sa1 :: lsa ⇒ Fset.is_empty (A T) (sa1 interS (Fset.Union _ lsa)) && all_distinct lsa
  end.

Fixpoint well_formed_q (sq : sql_query) :=
  match sq with
  | Sql_Table _ ⇒ true
  | Sql_Set _ sq1 sq2 ⇒ well_formed_q sq1 && well_formed_q sq2
  | Sql_Select s lsq f1 g f2 ⇒
    (all_distinct (map (fun x => sql_from_item_sort) x) lsq)
     && (forallb (fun x ⇒ match x with From_Item sq _ ⇒ well_formed_q sq end) lsq)
     && (well_formed_f f1) && (well_formed_f f2)
  end.

Lemma sql_query_to_alg_is_sound :
  well_sorted_sql_table →
    ∀env sq, well_formed_q sq = true ->
      eval_alg_query env (sql_query_to_alg basesort sq) =BE= eval_sql_query env sq.

Lemma alg_query_to_sql_is_sound :
  well_sorted_sql_table →
    ∀env q, eval_alg_query env q =BE= eval_sql_query env (alg_query_to_sql q).
```

Equivalence's theorems

```
Definition well_sorted_sql_table :=
  ∀ tbl t, t inBE (instance tbl) → labels t =S= basesort tbl.

Fixpoint all_distinct lsa :=
  match lsa with
    | nil ⇒ true
    | sa1 :: lsa ⇒ Fset.is_empty (A T) (sa1 interS (Fset.Union _ lsa)) && all_distinct lsa
  end.

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  | Sql_Set _ sq1 sq2 ⇒ well_formed_q sq1 && well_formed_q sq2
  | Sql_Select s lsq f1 g f2 ⇒
    (all_distinct (map (fun x => sql_from_item_sort) x) lsq)
     && (forallb (fun x ⇒ match x with From_Item sq _ ⇒ well_formed_q sq end) lsq)
     && (well_formed_f f1) && (well_formed_f f2)
  end.

Lemma sql_query_to_alg_is_sound :
  well_sorted_sql_table → (* cartesian product = natural join thanks to to well-formedness *)
    ∀ env sq, well_formed_q sq = true ->
      eval_alg_query env (sql_query_to_alg basesort sq) =BE= eval_sql_query env sq.

Lemma alg_query_to_sql_is_sound :
  well_sorted_sql_table →
    ∀ env q, eval_alg_query env q =BE= eval_sql_query env (alg_query_to_sql q).
```

Equivalence's theorems

```
Definition well_sorted_sql_table :=
  ∀tbl t, t inBE (instance tbl) → labels t =S= basesort tbl.

Fixpoint all_distinct lsa :=
  match lsa with
    | nil ⇒ true
    | sa1 :: lsa ⇒ Fset.is_empty (A T) (sa1 interS (Fset.Union _ lsa)) && all_distinct lsa
  end.

Fixpoint well_formed_q (sq : sql_query) :=
  match sq with
  | Sql_Table _ ⇒ true
  | Sql_Set _ sq1 sq2 ⇒ well_formed_q sq1 && well_formed_q sq2
  | Sql_Select s lsq f1 g f2 ⇒
    (all_distinct (map (fun x => sql_from_item_sort) x) lsq)
     && (forallb (fun x ⇒ match x with From_Item sq _ ⇒ well_formed_q sq end) lsq)
     && (well_formed_f f1) && (well_formed_f f2)
  end.

Lemma sql_query_to_alg_is_sound :
  well_sorted_sql_table → (* cartesian product = natural join thanks to to well-formedness *)
    ∀env sq, well_formed_q sq = true ->
      eval_alg_query env (sql_query_to_alg basesort sq) =BE= eval_sql_query env sq.

Lemma alg_query_to_sql_is_sound :
  well_sorted_sql_table → (* cartesian product = natural join thanks to to fresh names *)
    ∀env q, eval_alg_query env q =BE= eval_sql_query env (alg_query_to_sql q).
```

Lessons : Coq side

Modelling a real-life language

⇝ pushing Coq to the very limits

⇝ discovering some practical restrictions

with no theoretical reason

Lessons : Coq side

```
(* an abstracted version of formula's sharing between SQL queries and algebraic queries *)
Section FirstVersion.
Hypothesis A : Type.
Inductive (* first version of formula *) b : Type :=  B : A → b
with (* first version of sql query *) mut : Type :=  M : b → mut.
End FirstVersion.

Inductive (* first tentative version of algbraic query *) x : Type :=  X : (b x) → x.
(* Error: Non strictly positive occurrence of "x" in "b x → x". *)

Section SecondVersion.
Hypothesis A : Type.
Inductive (* new version formula *) b' : Type :=  B' : A → b'.
Inductive (* new version of sql query *) mut' : Type :=  M' : b' → mut'.
End SecondVersion.

Inductive (* new style algebraic query *) x1 : Type := X1 : (b' x1) → x1.

(*
x1 is defined
x1_rect is defined
x1_ind is defined
x1_rec is defined
*)
```

Lessons : DB side

first version : set-semantics ⇝ second version: bag-semantics
technical, not a problem

NULL's at expression level, absorbing elements
at formula level, use 3-valued logic...
not so difficult

real difficulty
complex expressions and nested and correlated queries
environments management

remains to be done:
`outer, inner join` (syntactic sugar) `order by, windows, rank`,
recursive queries
`like` handling regular expressions for strings
more data types: `date`

Epilogue

Data centric languages : a fantastic bestiary

NoSQL, Cassandra, MongoDB, Neo4j, etc weird

SQL purposely not Turing complete
$\rightsquigarrow$ overtime, new primitives and features:
aggregates, nested / correlated queries, functions, `NULL`'s
$\rightsquigarrow$ uncontrolled interactions
$\rightsquigarrow$ departing from its elegant theoretical foundation
$\rightsquigarrow$ pay tribute to pioneers: Codd, Chamberlin, Boyce

use Coq to design new languages
$\rightsquigarrow$ completely formalised, clear and well-understood semantics